# 3.5. Ispitivanje toka funkcije 2

20. 11. 2020.

Fotografija: Paul Einerhand

# Zadatak 32(a)

Ispitajte tok i skicirajte graf funkcije $f(x) := \dfrac{1}{x^2 + x + 1}$.

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| | $-\infty$ | | $-1$ | | $-\frac{1}{2}$ | | $0$ | | $+\infty$ |
|---|---|---|---|---|---|---|---|---|---|
| $f$ | $\times$ | | | | | | | | $\times$ |
| $f'$ | $\times$ | | | | $0$ | | | | $\times$ |
| $f''$ | $\times$ | | $0$ | | | | $0$ | | $\times$ |
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|---|---|---|---|---|---|---|---|---|---|
| $f$ | $\times$ | $+$ | | $+$ | | $+$ | | $+$ | $\times$ |
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| $f$ | $\times$ | $+$ | | $+$ | | $+$ | | $+$ | $\times$ |
| $f'$ | $\times$ | $+$ | | $+$ | $0$ | $-$ | | $-$ | $\times$ |
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| $f$ | $\times$ | $\nearrow$ $+$ | | $\nearrow$ $+$ | | $\searrow$ $+$ | | $\searrow$ $+$ | $\times$ |
| $f'$ | $\times$ | $+$ | | $+$ | $0$ | $-$ | | $-$ | $\times$ |
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| $f$ | $\times$ | $\nearrow$ + $\cup$ | | $\nearrow$ + $\cap$ | | $\searrow$ + $\cap$ | | $\searrow$ + $\cup$ | $\times$ |
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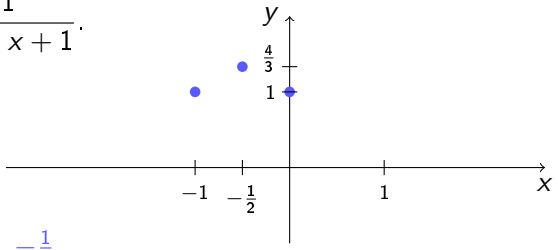


glob. max

| | $-\infty$ | | $-1$ | | $-\frac{1}{2}$ | | $0$ | | $+\infty$ |
|---|---|---|---|---|---|---|---|---|---|
| $f$ | × | ↗ + ∪ | | ↗ + ∩ | | ↘ + ∩ | | ↘ + ∪ | × |
| $f'$ | × | + | | + | 0 | − | | − | × |
| $f''$ | × | + | 0 | − | | − | 0 | + | × |
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| $f'$ | $\times$ | + | | + | 0 | − | | − | $\times$ |
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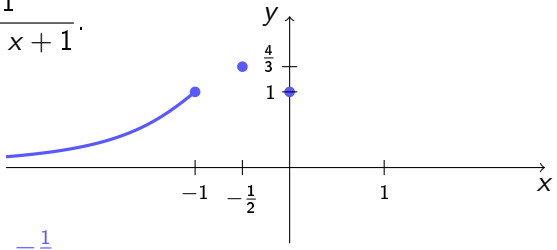

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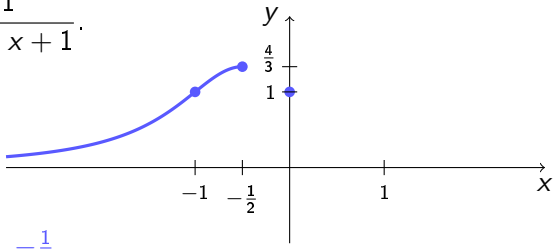


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| $f$ | $\times$ | $\nearrow$ + $\cup$ | | $\nearrow$ + $\cap$ | | $\searrow$ + $\cap$ | | $\searrow$ + $\cup$ | $\times$ |
| $f'$ | $\times$ | + | | + | 0 | − | | − | $\times$ |
| $f''$ | $\times$ | + | 0 | − | | − | 0 | + | $\times$ |
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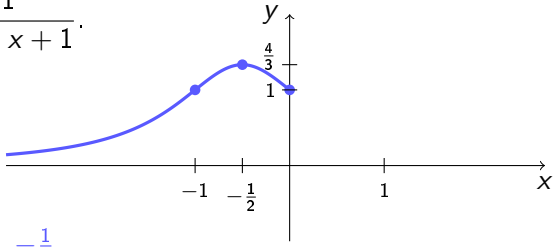


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| $f'$ | $\times$ | + | | + | 0 | − | | − | $\times$ |
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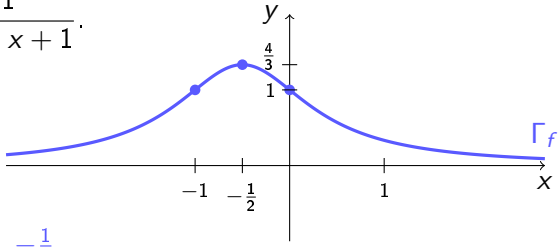


*Rješenje.*

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2. $f$ nije ni parna ni neparna ni periodična.
3. $f$ nema nultočaka.
4. $f'(x) = -\frac{2x+1}{(x^2+x+1)^2}$ $\rightsquigarrow$ Stacionarne točke: $-\frac{1}{2}$.
5. $f''(x) = \frac{6x(x+1)}{(x^2+x+1)^3}$ $\rightsquigarrow$ $f''(x) = 0 \Leftrightarrow x \in \{-1, 0\}$.
6. $x \to \pm\infty \Rightarrow \frac{1}{x^2+x+1} \to 0$ $\rightsquigarrow$ $y = 0$ je horizontalna asimptota.
7. glob. max

| | $-\infty$ | | $-1$ | | $-\frac{1}{2}$ | | $0$ | | $+\infty$ |
|---|---|---|---|---|---|---|---|---|---|
| $f$ | $\times$ | $\nearrow$ | $+$ | $\cup$ | $\nearrow$ | $+$ | $\cap$ | $\searrow$ | $+$ | $\cap$ | $\searrow$ | $+$ | $\cup$ | $\times$ |
| $f'$ | $\times$ | | $+$ | | | $+$ | $0$ | | $-$ | | | $-$ | | $\times$ |
| $f''$ | $\times$ | | $+$ | $0$ | | $-$ | | | $-$ | $0$ | | $+$ | | $\times$ |
| $f(x)$ | $0 \leftarrow$ | | | $1$ | | | $\frac{4}{3}$ | | | $1$ | | | $\to 0$ | |

# Zadatak 32(b)

Isp. tok i skic. graf fje $f : \langle 0, 2\pi \rangle \setminus \{\pi\} \to \mathbb{R}$, $f(x) := \dfrac{1}{\sin x}$.

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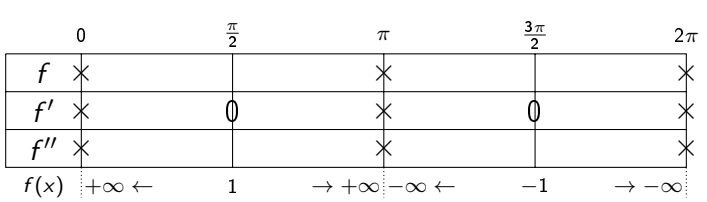
7. 

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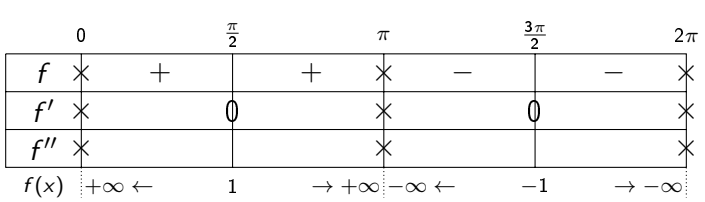
7. 

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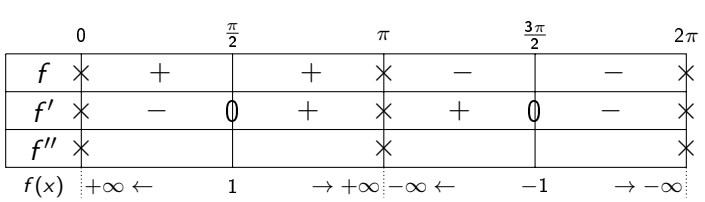
7. 

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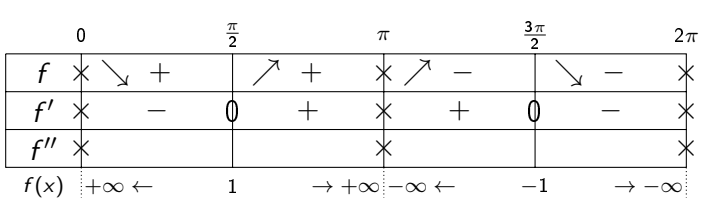
7. 

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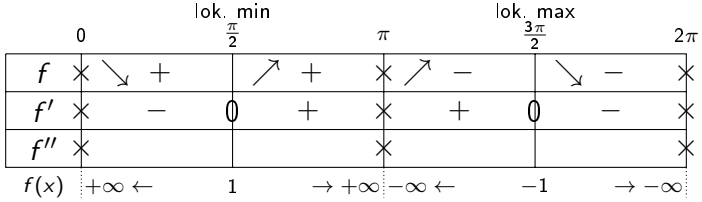
7. 

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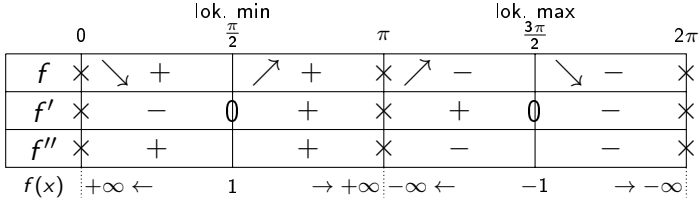
7. 

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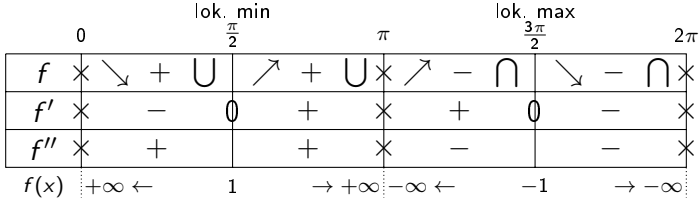
7. 

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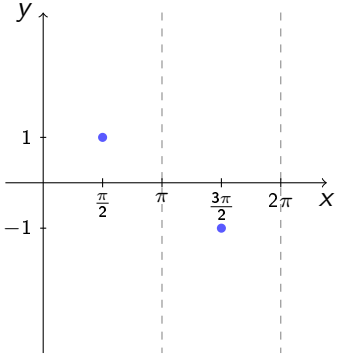
5. $f''(x) = \frac{\sin^2 x + 2\cos^2 x}{\sin^3 x}$. $\rightsquigarrow$ $f''(x) \neq 0$ za sve $x \in \mathbb{R}$.
6. $x \to 0+ \ \Rightarrow\ \frac{1}{\sin x} \to +\infty$
   $x \to \pi\mp \ \Rightarrow\ \frac{1}{\sin x} \to \pm\infty$
   $x \to 2\pi- \ \Rightarrow\ \frac{1}{\sin x} \to -\infty$.
   $\rightsquigarrow$ V. a.: $x = 0$, $x = \pi$, $x = 2\pi$.



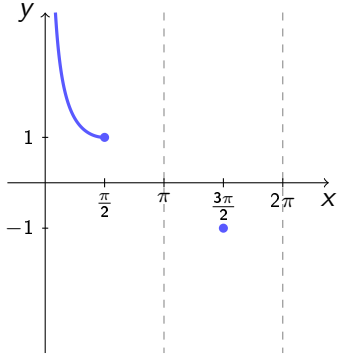

❼

| | 0 | | $\frac{\pi}{2}$ (lok. min) | | $\pi$ | | $\frac{3\pi}{2}$ (lok. max) | | $2\pi$ |
|---|---|---|---|---|---|---|---|---|---|
| $f$ | × | ↘ | + | ∪ | × | ↗ | + | ∪ | × | ↗ | − | ∩ | × | ↘ | − | ∩ | × |
| $f'$ | × | − | 0 | + | × | + | 0 | − | × |
| $f''$ | × | + | | + | × | − | | − | × |
| $f(x)$ | $+\infty \leftarrow$ | | 1 | | $\to +\infty$ | $-\infty \leftarrow$ | | $-1$ | | $\to -\infty$ |

# Zadatak 32(b)

Isp. tok i skic. graf fje $f : \langle 0, 2\pi\rangle \setminus \{\pi\} \to \mathbb{R}$, $f(x) := \dfrac{1}{\sin x}$.

*Rješenje.*

1. $D = \langle 0, \pi\rangle \cup \langle \pi, 2\pi\rangle$. $\rightsquigarrow$ Rubovi domene: $0$, $\pi$, $2\pi$.
2. $f$ nije ni parna ni neparna ni periodična.
3. $f$ nema nultočaka.
4. $f'(x) = -\frac{1}{\sin^2 x} \cdot \cos x$. $\rightsquigarrow$ Stacionarne točke: $\frac{\pi}{2}$, $\frac{3\pi}{2}$.
5. $f''(x) = \frac{\sin^2 x + 2\cos^2 x}{\sin^3 x}$. $\rightsquigarrow$ $f''(x) \neq 0$ za sve $x \in \mathbb{R}$.
6. $x \to 0+ \;\Rightarrow\; \frac{1}{\sin x} \to +\infty$
   $x \to \pi\mp \;\Rightarrow\; \frac{1}{\sin x} \to \pm\infty$
   $x \to 2\pi- \;\Rightarrow\; \frac{1}{\sin x} \to -\infty$.
   $\rightsquigarrow$ V. a.: $x = 0$, $x = \pi$, $x = 2\pi$.
8. 


7. lok. min $\frac{\pi}{2}$, lok. max $\frac{3\pi}{2}$

| $x$ | $0$ | | $\frac{\pi}{2}$ | | $\pi$ | | $\frac{3\pi}{2}$ | | $2\pi$ |
|---|---|---|---|---|---|---|---|---|---|
| $f$ | $\times$ | $\searrow$ | $+$ | $\cup$ | $\nexists$ | $\nearrow$ | $+$ | $\cup$ | $\nexists$ | $\nearrow$ | $-$ | $\cap$ | $\nexists$ | $\searrow$ | $-$ | $\cap$ | $\times$ |
| $f'$ | $\times$ | $-$ | $0$ | $+$ | $\nexists$ | $+$ | $0$ | $-$ | $\times$ |
| $f''$ | $\times$ | $+$ | | $+$ | $\nexists$ | $-$ | | $-$ | $\times$ |
| $f(x)$ | $+\infty \leftarrow$ | | $1$ | | $\to +\infty$ | $-\infty \leftarrow$ | | $-1$ | | $\to -\infty$ |

# Zadatak 32(b)

Isp. tok i skic. graf fje $f : \langle 0, 2\pi\rangle \setminus \{\pi\} \to \mathbb{R}$, $f(x) := \dfrac{1}{\sin x}$. ❽

*Rješenje.*

1. $D = \langle 0, \pi\rangle \cup \langle \pi, 2\pi\rangle$. $\rightsquigarrow$ Rubovi domene: $0$, $\pi$, $2\pi$.
2. $f$ nije ni parna ni neparna ni periodična.
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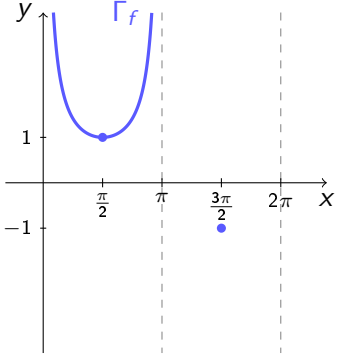
   $\rightsquigarrow$ V. a.: $x = 0$, $x = \pi$, $x = 2\pi$.




❼

| | 0 | | $\frac{\pi}{2}$ (lok. min) | | $\pi$ | | $\frac{3\pi}{2}$ (lok. max) | | $2\pi$ |
|---|---|---|---|---|---|---|---|---|---|
| $f$ | × | ↘ | + ∪ | ↗ | + ∪ | × ↗ | − ∩ | ↘ | − ∩ × |
| $f'$ | × | − | 0 | + | × | + | 0 | − | × |
| $f''$ | × | + | | + | × | − | | − | × |
| $f(x)$ | $+\infty \leftarrow$ | | 1 | $\to +\infty$ | $-\infty \leftarrow$ | | $-1$ | | $\to -\infty$ |

# Zadatak 32(b)

Isp. tok i skic. graf fje $f : \langle 0, 2\pi\rangle \setminus \{\pi\} \to \mathbb{R}$, $f(x) := \dfrac{1}{\sin x}$. ❽

*Rješenje.*

1. $D = \langle 0, \pi\rangle \cup \langle \pi, 2\pi\rangle$. $\rightsquigarrow$ Rubovi domene: $0$, $\pi$, $2\pi$.
2. $f$ nije ni parna ni neparna ni periodična.
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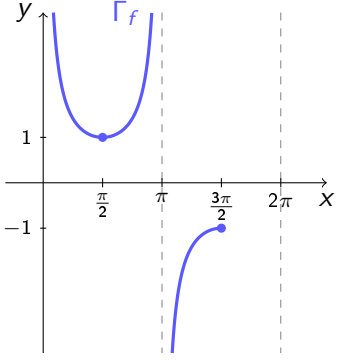
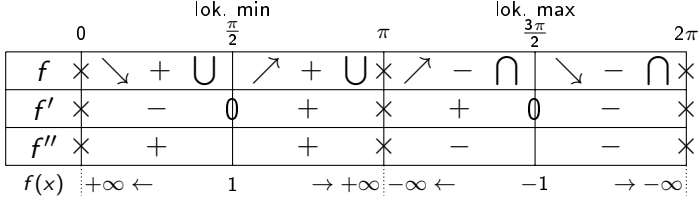
   $\rightsquigarrow$ V. a.: $x = 0$, $x = \pi$, $x = 2\pi$.




❼



# Zadatak 32(b)

Isp. tok i skic. graf fje $f : \langle 0, 2\pi\rangle \setminus \{\pi\} \to \mathbb{R},\ f(x) := \dfrac{1}{\sin x}$.

*Rješenje.*

1. $D = \langle 0, \pi\rangle \cup \langle \pi, 2\pi\rangle$. $\rightsquigarrow$ Rubovi domene: $0$, $\pi$, $2\pi$.
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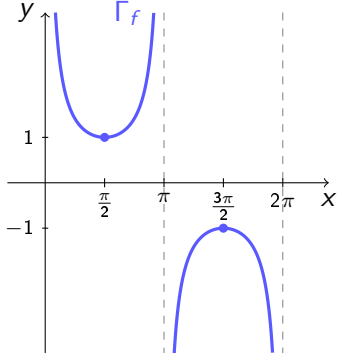
   $\rightsquigarrow$ V. a.: $x = 0$, $x = \pi$, $x = 2\pi$.
7. 

| | 0 | | lok. min $\frac{\pi}{2}$ | | $\pi$ | | lok. max $\frac{3\pi}{2}$ | | $2\pi$ |
|---|---|---|---|---|---|---|---|---|---|
| $f$ | $\times$ | $\searrow$ | $+$ | $\cup$ | $\nexists$ | $\nearrow$ | $+$ | $\cup$ | $\times$ | $\nearrow$ | $-$ | $\cap$ | $\nexists$ | $\searrow$ | $-$ | $\cap$ | $\times$ |
| $f'$ | $\times$ | $-$ | $0$ | $+$ | $\times$ | $+$ | $0$ | $-$ | $\times$ | | | | | | | | |
| $f''$ | $\times$ | $+$ | | $+$ | $\times$ | $-$ | | $-$ | $\times$ | | | | | | | | |
| $f(x)$ | $+\infty \leftarrow$ | | $1$ | | $\to +\infty$ | $-\infty \leftarrow$ | | $-1$ | | $\to -\infty$ | | | | | | | |

8. 

# Zadatak 32(c)

Ispitajte tok i skicirajte graf funkcije $f(x) := \dfrac{1}{\sin x}$.

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Ispitajte tok i skicirajte graf funkcije $f(x) := \dfrac{1}{\sin x}$.

*Rješenje.*

1. $D = \mathcal{D}_f = \mathbb{R} \setminus \{k\pi : k \in \mathbb{Z}\}$.

# Zadatak 32(c)

Ispitajte tok i skicirajte graf funkcije $f(x) := \dfrac{1}{\sin x}$.

*Rješenje.*

1. $D = \mathcal{D}_f = \mathbb{R} \setminus \{k\pi : k \in \mathbb{Z}\}$.
2. - $f(-x) = \frac{1}{\sin(-x)} = -\frac{1}{\sin x} = -f(x)$
   $\rightsquigarrow$ $f$ je neparna, nije parna.

# Zadatak 32(c)

Ispitajte tok i skicirajte graf funkcije $f(x) := \dfrac{1}{\sin x}$.

*Rješenje.*

1. $D = \mathcal{D}_f = \mathbb{R} \setminus \{k\pi : k \in \mathbb{Z}\}$.
2. - $f(-x) = \frac{1}{\sin(-x)} = -\frac{1}{\sin x} = -f(x)$
   $\rightsquigarrow$ $f$ je neparna, nije parna.
   - $f(x + 2\pi) = \frac{1}{\sin(x+2\pi)} = \frac{1}{\sin x} = f(x)$
   $\rightsquigarrow$ $f$ je $2\pi$-periodična.

# Zadatak 32(c)

Ispitajte tok i skicirajte graf funkcije $f(x) := \dfrac{1}{\sin x}$.

*Rješenje.*

1. $D = \mathcal{D}_f = \mathbb{R} \setminus \{k\pi : k \in \mathbb{Z}\}$.
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   - $f(-x) = \frac{1}{\sin(-x)} = -\frac{1}{\sin x} = -f(x)$
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   - $f(x + 2\pi) = \frac{1}{\sin(x+2\pi)} = \frac{1}{\sin x} = f(x)$
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! Zbog $2\pi$-periodičnosti funkcije $f$ dovoljno je ispitati njen tok na nekom dijelu domene duljine $2\pi$, a to smo napravili u Zad. 32(b).

# Zadatak 32(c)

Ispitajte tok i skicirajte graf funkcije $f(x) := \dfrac{1}{\sin x}$.

*Rješenje.*

1. $D = \mathcal{D}_f = \mathbb{R} \setminus \{k\pi : k \in \mathbb{Z}\}$.
2. 
   - $f(-x) = \frac{1}{\sin(-x)} = -\frac{1}{\sin x} = -f(x)$
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! Zbog $2\pi$-periodičnosti funkcije $f$ dovoljno je ispitati njen tok na nekom dijelu domene duljine $2\pi$, a to smo napravili u Zad. 32(b).

8.